Section 5.1 Polynomial functions

1) $f(x) = (x-3)^2(x+1)$

a) $(3,0)$ *multiplicity* 2 $(even)$ $(-1,0)$ *multiplicity* 1 (odd)

b) *touches* $x-$*axis at* $(3,0)$ *crosses the* $x-$*axis at* $(1,0)$

c) maximum 2 turning points

d) Sketch a graph and approximate the turning points, also label the x-intercepts

e) *resembles* $f(x) = x^3$

f) *increasing* $(-\infty, 0.33) \cup (3, \infty)$ *decreasing* $(0.33, 3)$

1a) $(x-3)^2(x+1)=0$

$x-3=0$ $\quad x+1=0$

$x=3$ $\quad x=-1$

$(3,0)$ $(-1,0)$

1b) Even exponent touches
Odd exponent crosses

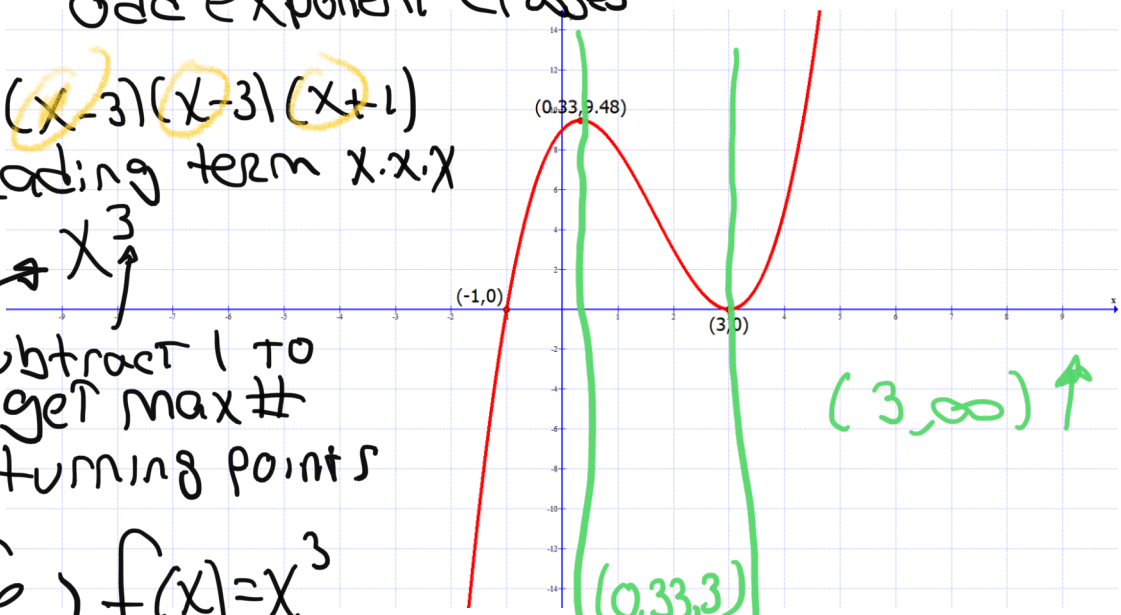
1c) $(x-3)(x-3)(x+1)$

leading term $x \cdot x \cdot x$

x^3

Subtract 1 to get max # turning points

1e) $f(x)=x^3$



$(3,\infty)$ ↑

$(0.33,3)$ ↓

$(-\infty, 0.33)$ ↑

3) $f(x) = (x-3)^3(x+4)$

a) $(3,0)\ multiplicity\ 3\ (odd)\ (-4,0)\ multiplicity\ 1\ (odd)$

b) $crosses\ at\ (3,0)\ crosses\ at\ (-4,0)$ (Each has odd multiplicity)

c) maximum 3 turning points

d) Sketch a graph and approximate the turning points, also label the x-intercepts

e) $resembles\ f(x) = x^4$

f) $increasing\ (-2.25,3) \cup (3,\infty)\ decreasing(-\infty,-2.25)$

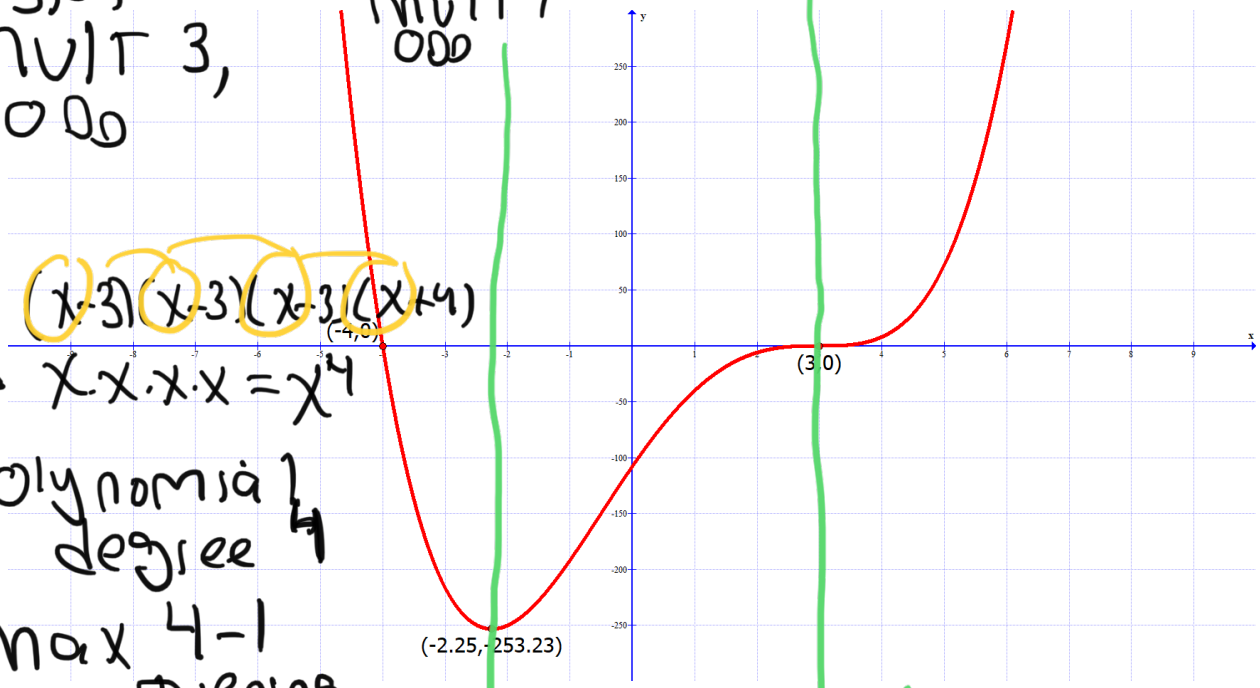
3a $(x-3)^3(x+4)=0$

$x-3=0$ $x+4=0$

$x=3$ $x=-4$

$(3,0)$ $(-4,0)$

mult 3, odd mult 1 odd



3c) $(x-3)(x-3)(x-3)(x+4)$

$x \cdot x \cdot x \cdot x = x^4$

Polynomial degree 4

max 4-1 turning points

3e) $f(x) = x^4$

$(-\infty, -2.25)$ ↑

$(-2.25, 3)$ ↑

$(3, \infty)$ ↑

5) $f(x) = (x+3)(x-3)(3x+21)$

a) $(-3,0)$ $multplicity\ 1\ (odd)$

$(3,0)$ $multiplicity\ 1\ (odd)$

$(-7,0)$ $multiplicity\ 1\ (odd)$

b) $crosses\ at\ each\ x-intercept$ (since each odd multiplicity)

c) $maximu\ 2\ turning\ points$

d) Sketch a graph and approximate the turning points, also label the x-intercepts

e) $resembles\ f(x) = 3x^3$

f) $increasing\ (-\infty, -5.24) \cup (0.57, \infty)\ decreasing\ (-5.24, 0.57)$

(a)

$(x+3)(x-3)(3x+21)=0$

$x+3=0$ $x-3=0$ $3x+21=0$

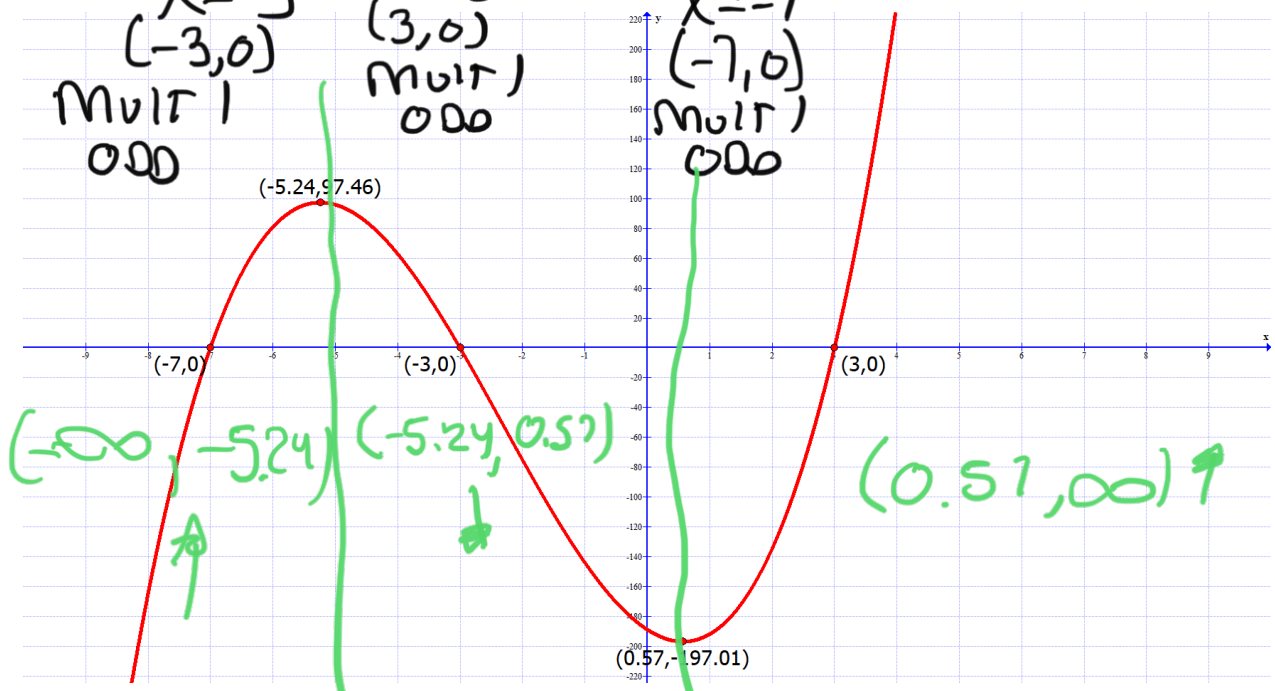
$x=-3$ $x=3$ $3x=-21$

$x=-7$

$(-3,0)$ $(3,0)$ $(-7,0)$

Mult 1 Mult 1 Mult 1

odd odd odd



1c) $(x\ )(x\ )(3x$

$-1 \cdot x \cdot x \cdot 3x = 3x^3$

degree 3 max $3-1=2$ turning points

1e) Resembles $f(x) = 3x^3$

7) $f(x) = (x+3)^2(2x-10)$

a) $(-3,0)$ *multiplicity* 2 *(even)* $(5,0)$ *multiplicity* 1 *(odd)*

b) $(-3,0)$ *touches* $(5,0)$ *crosses*

c) *maximum* 2 *turning points*

d) Sketch a graph and approximate the turning points, also label the x-intercepts

e) *resembles* $f(x) = 2x^3$

f) *increasing* $(-\infty,-3) \cup (2.33,\infty)$ *decreasing* $(-3,2.33)$

a) $(x+3)^2(2x-10)=0$

$x+3=0$ $\quad 2x-10=0$

$x=-3$ $\quad 2x=10$

$x=5$

$(-3,0)$ $\quad (5,0)$

mult 2 $\quad$ mult 1

Even $\quad$ Odd

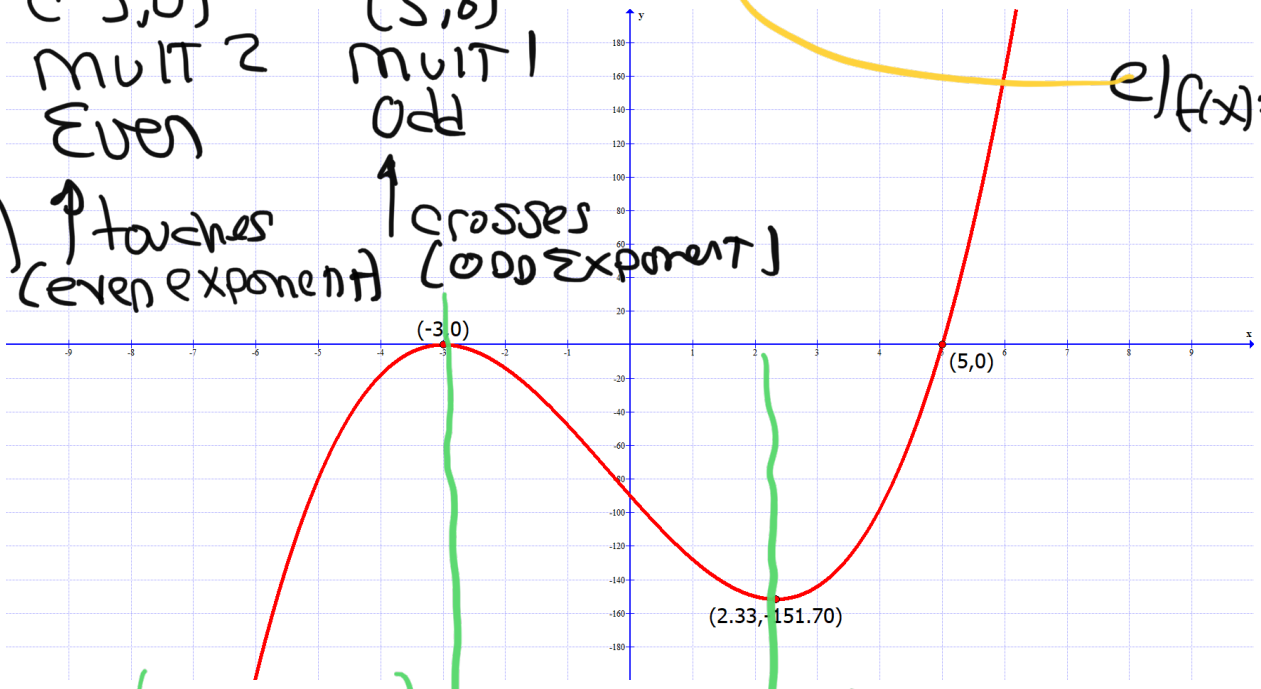
b) touches (even exponent) $\quad$ crosses (odd exponent)

c) $(x+3)(x+3)(2x-10)$

$x \cdot x \cdot 2x = 2x^3$

max $3-1=2$ turning points

e) $f(x)=2x^3$



$(-\infty,-3)$ $\quad (-3,2.33)$ $\quad (2.33,\infty)$

9) $f(x) = x^2 + 6x - 7$

a) $(-7,0)$ *multiplicity* 1 *(odd)* $(1,0)$ *multiplicity* 1 *(odd)*

b) *graph crosses the* $x-$*axis at both* $x-$*intercepts* (Since odd mult.)

c) *maximum* 1 *turning point*

d) Sketch a graph and approximate the turning points, also label the x-intercepts

e) *resembles* $f(x) = x^2$

f) *increasing* $(-3,\infty)$ *decreasing* $(-\infty,3)$

a) $x^2+6x-7=0$

$(x+7)(x-1)=0$

$x+7=0$ $\quad x-1=0$

$x=-7$ $\quad x=1$

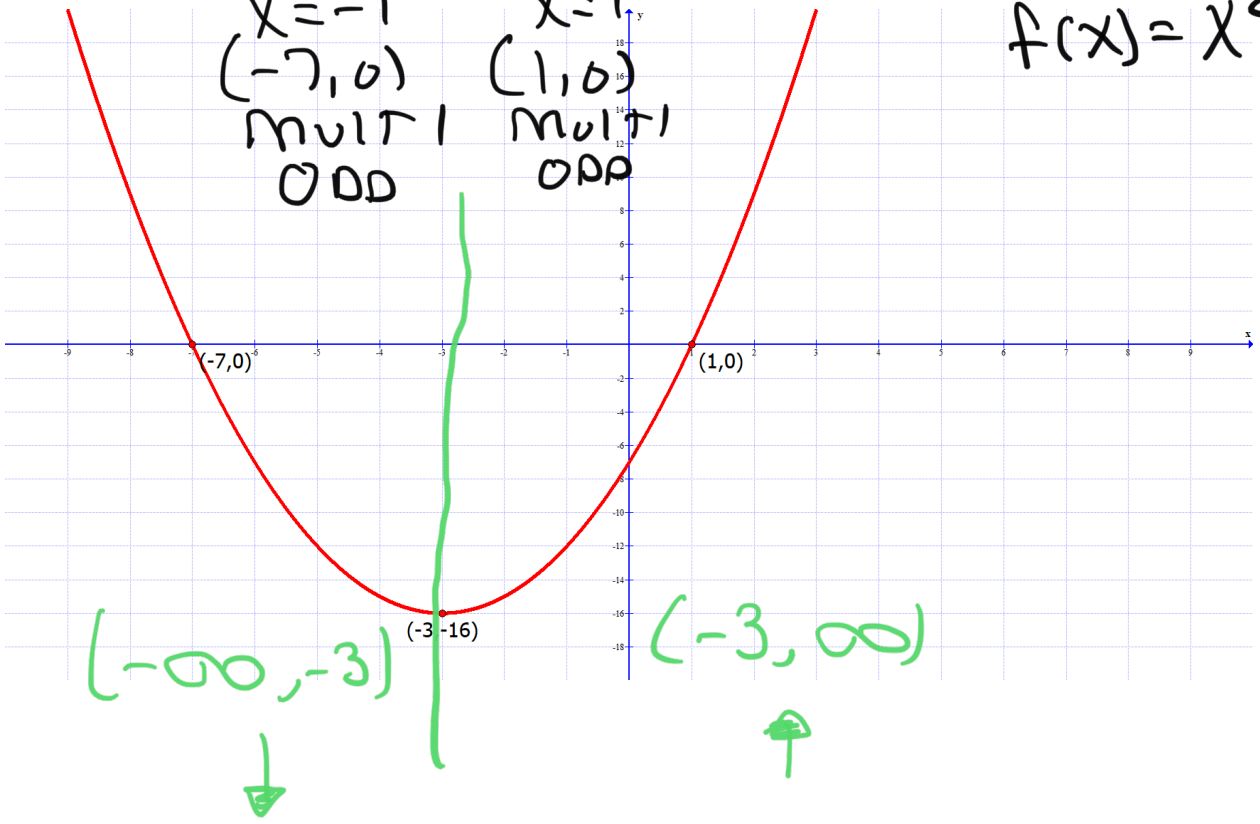
$(-7,0)$ $\quad (1,0)$

mult 1 $\quad$ mult 1

odd $\quad$ odd

c) max 2-1=1 Turning point

e) Resemble $f(x)=x^2$



11) $f(x) = x^2 - 4$

a) $(-2,0)\ multiplicity\ 1\ (odd)\ \ (2,0)\ multiplicity\ 1\ (odd)$

b) $graph\ crosses\ the\ x-axis\ at\ both\ x-intercepts$ (Both odd mult)

c) $maximum\ 1\ turning\ point$

d) Sketch a graph and approximate the turning points, also label the x-intercepts

e) $resembles\ f(x) = x^2$

f) $increasing\ (0,\infty)\ \ decreasing\ \ (-\infty,0)$

11a) $x^2 - 4 = 0$

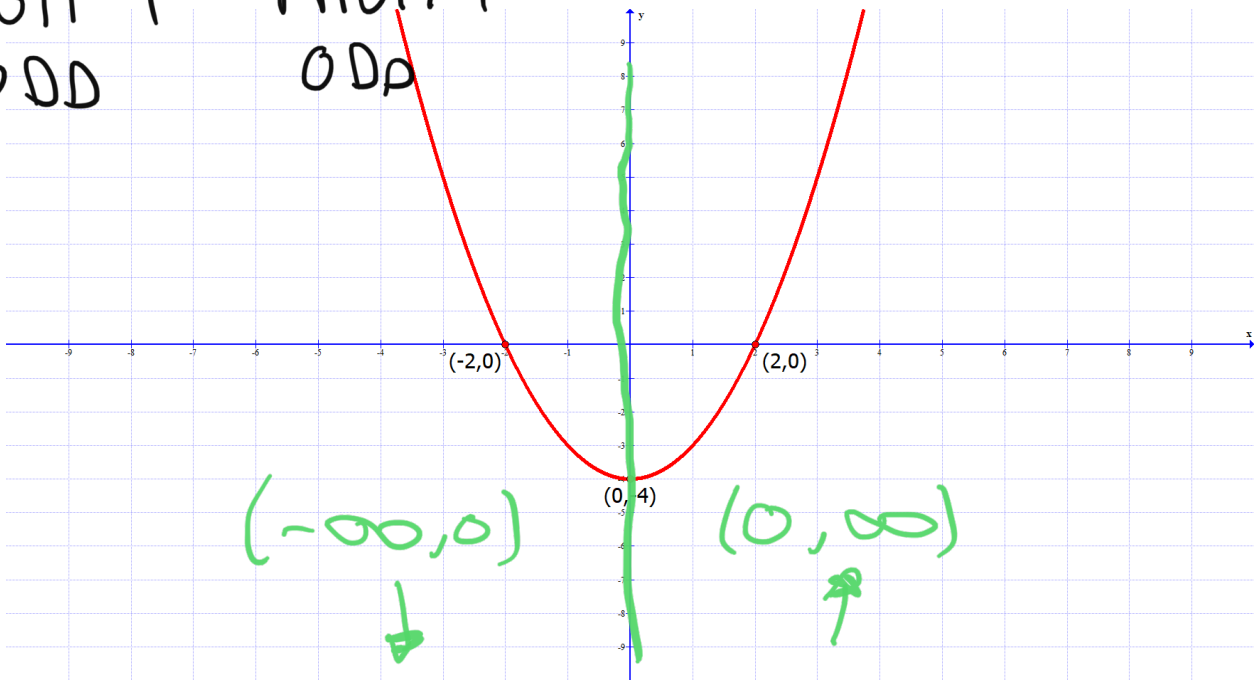
$(x+2)^1(x-2)^1 = 0$

$x = -2$ Mult 1 odd

$x = 2$ Mult 1 odd

c) Max 2-1=1 Turning point

e) Resembles $f(x) = x^2$



$(-\infty, 0)$ ↓ $(0, \infty)$ ↑

13) $f(x) = -3x^3 - 3x^2 + 18x$

a) $(-3,0)$ $multplicity\ 1\ (odd)$

$(0,0)$ $multiplicity\ 1\ (odd)$

$(2,0)$ $multiplicity\ 1\ (odd)$

b) $crosses\ at\ each\ x-intercept$ (All odd mult)

c) $maximu\ 2\ turning\ points$

d) Sketch a graph and approximate the turning points, also label the x-intercepts

e) $resembles\ f(x) = -3x^3$

f) $increasing\ (-1.78,1.12)$ $decreasing\ (-\infty,-1.78) \cup (1.12,\infty)$

a) $-3x^3 - 3x^2 + 18x = 0$

$-3x(x^2 + x - 6) = 0$

$-3x(x+3)(x-2) = 0$

$x = 0$ $x = -3$ $x = 2$

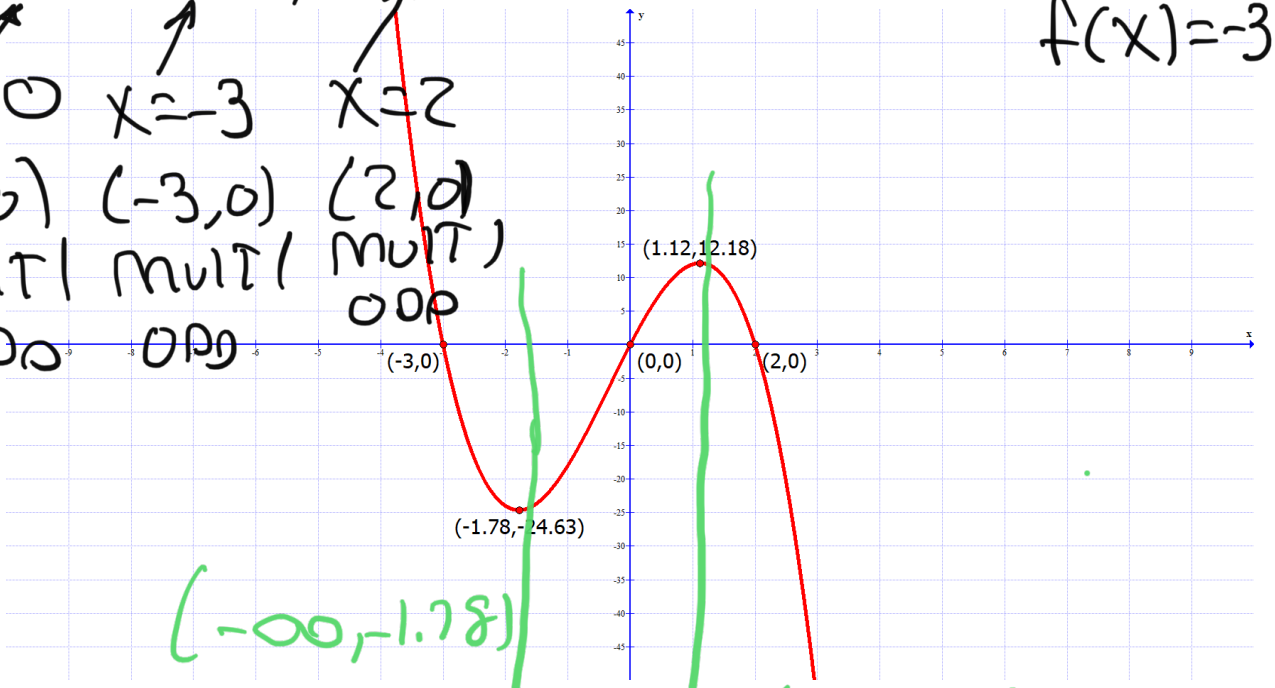
$(0,0)$ $(-3,0)$ $(2,0)$

Mult 1 Mult 1 Mult 1

odd odd odd

b) max 3-1=2 Turning points

c) Resembles $f(x) = -3x^3$



$(-\infty,-1.78)$ $(-1.78,1.12)$ $(1.12,\infty)$

15) $f(x) = 3x^4 - 12x^2$

a) $(-2,0)$ *multiplicity* 1 (odd) $(0,0)$ *multiplicity* 2 $(even)$

$(2,0)$ *multiplicity* 1 (odd)

b) *crosses at* $(2,0)$ *and* $(-2,0)$ *touches at* $(0,0)$

c) *maximum* 3 *turning points*

d) Sketch a graph and approximate the turning points, also label the x-intercepts

e) *resembles* $3x^4$

f) *increasing* $(-1.41,0) \cup (1.41,\infty)$ *decreasing* $(-\infty,-1.41) \cup (0,1.41)$

(a) $3x^4 - 12x^2 = 0$

$3x^2(x^2-4) = 0$

$3x^2(x+2)(x-2) = 0$

$x=0$ $x=-2$ $x=2$

$(0,0)$ $(-2,0)$ $(2,0)$

Mult 2 Mult 1 Mult 1

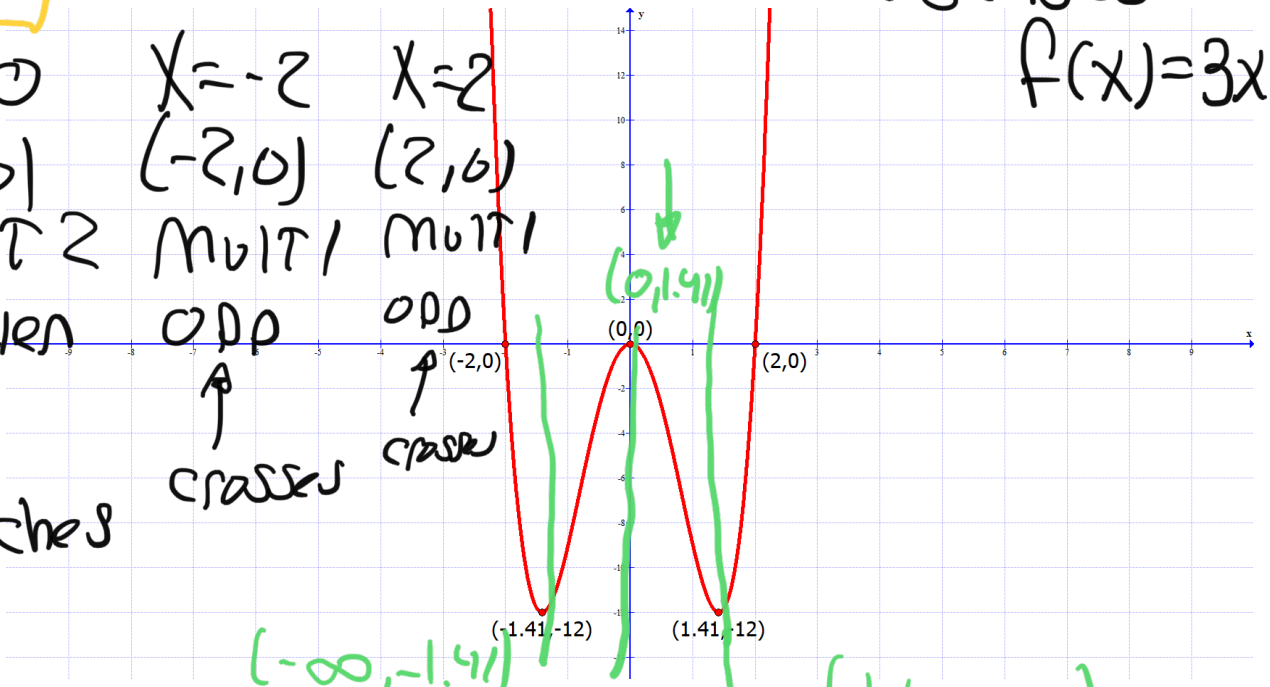
even odd odd

(b) Touches crosses crosses

(c) max 4-1=3

Turning points

e) Resembles $f(x)=3x^4$

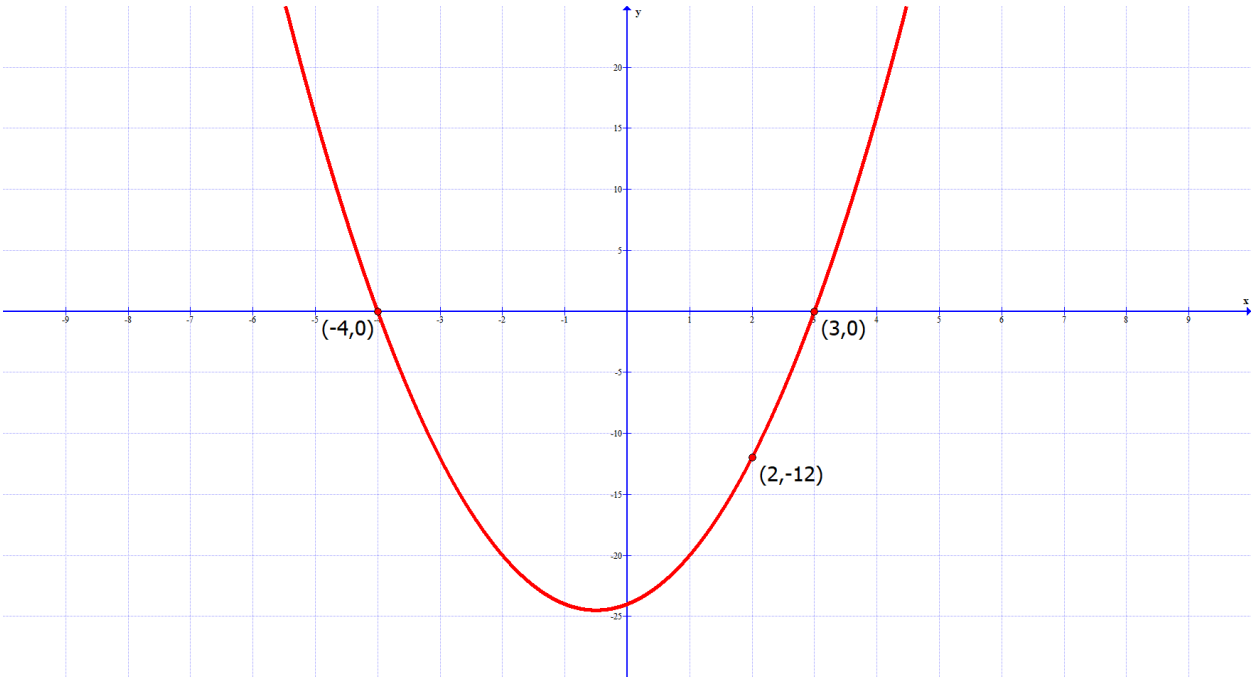


$(-\infty,-1.41)$ $(-1.41,0)$ $(1.41,\infty)$

17) x-intercepts: (3,0), (-4,0); point (2, -12)

$$f(x) = a(x-3)(x-(-4))$$
$$f(x) = a(x-3)(x+4)$$
$$-12 = a(2-3)(2+4)$$
$$-12 = a(-1)(6)$$
$$-12 = -6a$$
$$2 = a$$

Graph of $f(x) = 2(x-3)(x+4)$



$f(x) = 2(x-3)(x+4)$

19) x-intercepts: (-3,0), (4,0); point (5, -16)

$$f(x) = a(x-(-3))(x-4)$$

$$f(x) = a(x+3)(x-4)$$

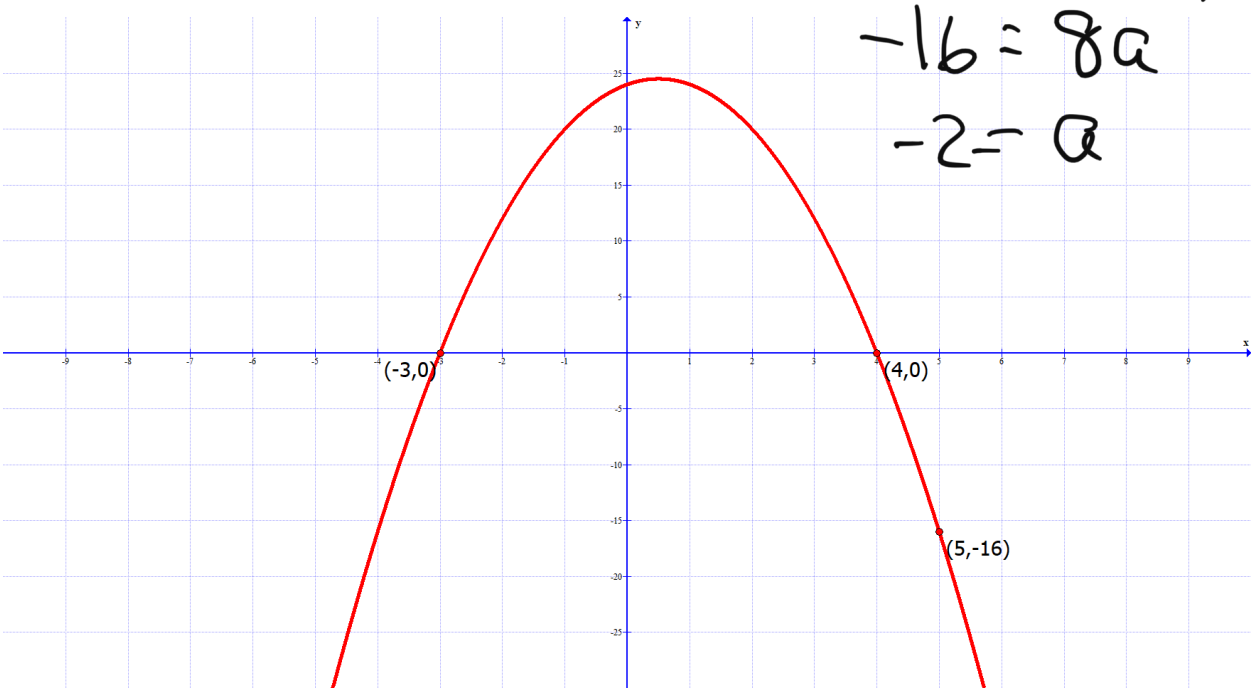
$$-16 = a(5+3)(5-4)$$

$$-16 = a(8)(1)$$

$$-16 = 8a$$

$$-2 = a$$

Graph of $f(x) = -2(x-4)(x+3)$



$f(x) = -2(x-4)(x+3)$

21) x-intercepts: (3,0) multiplicity 2, (-4,0); point (4, 16)

$f(x) = a(x-3)^2(x+4)$

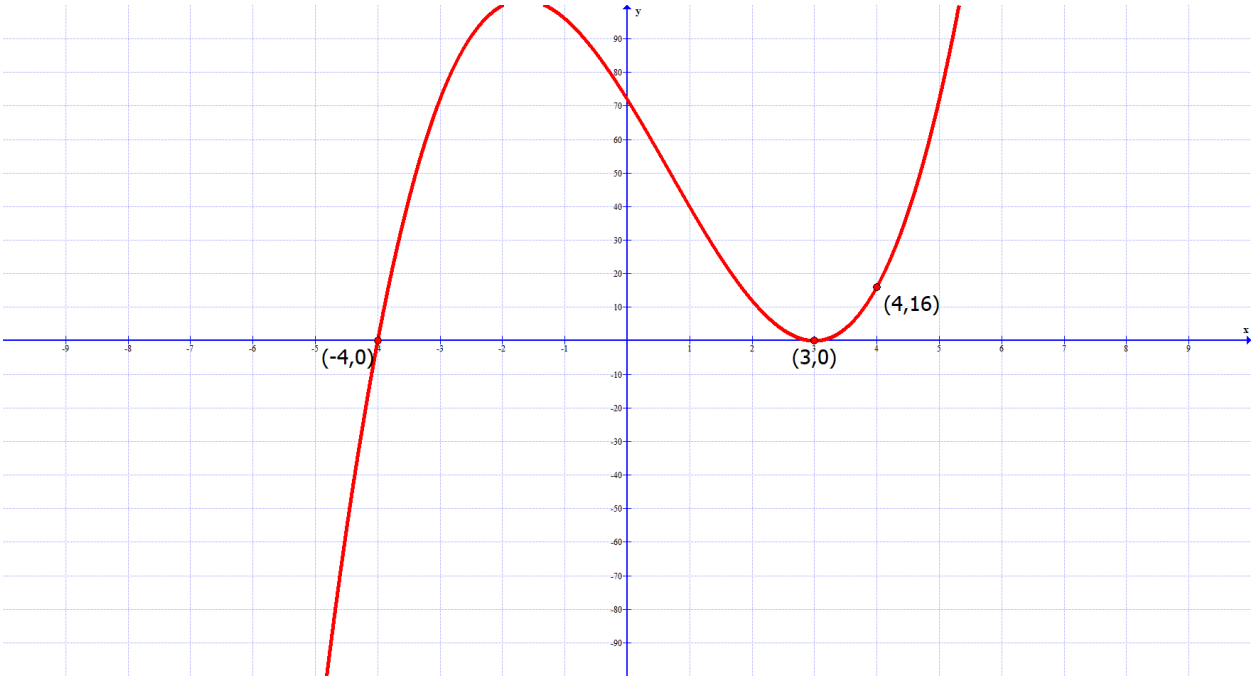
$16 = a(4-3)^2(4+4)$

$16 = a(1)^2(8)$

$16 = 8a$

$2 = a$

Graph of $f(x) = 2(x+4)(x-3)^2$



$f(x) = 2(x+4)(x-3)^2$

(23)

$$f(x) = a(x+3)(x-4)^2$$

$$-24 = a(3+3)(3-4)^2$$

$$-24 = a(6)(-1)^2$$

$$-24 = a(6)(1)$$

$$-24 = 6a$$

$$-4 = a$$

$$f(x) = -4(x+3)(x-4)^2$$